Cambridge Assessment International Education

Cambridge International AS & A Level

CANDIDATE NAME

CENTRE NUMBER

CANDIDATE NUMBER

MATHEMATICS **9709/13**

Paper 1 Pure Mathematics 1 **May/June 2021**

1 hour 50 minutes

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 75.
- The number of marks for each question or part question is shown in brackets [ ].

This document has **20** pages. Any blank pages are indicated.

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1 A curve with equation $y = f(x)$ is such that $f'(x) = 6x^2 - \frac{8}{x^2}$. It is given that the curve passes through the point $(2, 7)$.

Find $f(x)$. [3]

2 The function f is defined by $f(x) = \frac{1}{3}(2x - 1)^{\frac{3}{2}} - 2x$ for $\frac{1}{2} < x < a$. It is given that f is a decreasing function.

Find the maximum possible value of the constant a. [4]

3 A line with equation $y = mx - 6$ is a tangent to the curve with equation $y = x^2 - 4x + 3$.

Find the possible values of the constant m, and the corresponding coordinates of the points at which the line touches the curve. [6]

4 **(a)** Show that the equation

$$\frac{\tan x + \sin x}{\tan x - \sin x} = k,$$

where k is a constant, may be expressed as

$$\frac{1 + \cos x}{1 - \cos x} = k.$$ [2]

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(b) Hence express $\cos x$ in terms of k. [2]

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(c) Hence solve the equation $\frac{\tan x + \sin x}{\tan x - \sin x} = 4$ for $-\pi < x < \pi$. [2]

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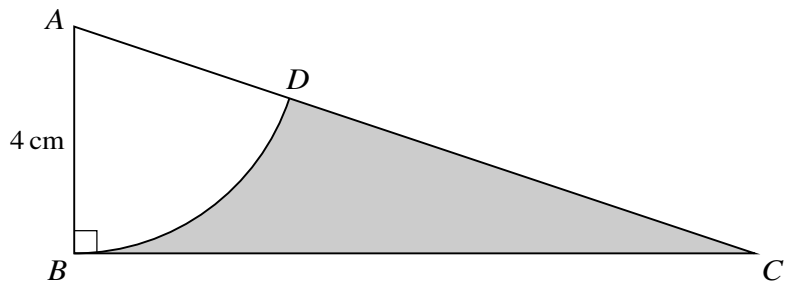

The diagram shows a triangle ABC, in which angle $ABC = 90°$ and $AB = 4$ cm. The sector ABD is part of a circle with centre A. The area of the sector is $10\,\text{cm}^2$.

(a) Find angle BAD in radians. [2]

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(b) Find the perimeter of the shaded region. [4]

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6 Functions f and g are both defined for $x \in \mathbb{R}$ and are given by

$$f(x) = x^2 - 2x + 5,$$
$$g(x) = x^2 + 4x + 13.$$

(a) By first expressing each of $f(x)$ and $g(x)$ in completed square form, express $g(x)$ in the form $f(x + p) + q$, where p and q are constants. [4]

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(b) Describe fully the transformation which transforms the graph of $y = f(x)$ to the graph of $y = g(x)$. [2]

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7 **(a)** Write down the first four terms of the expansion, in ascending powers of x, of $(a - x)^6$. [2]

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(b) Given that the coefficient of x^2 in the expansion of $\left(1 + \frac{2}{ax}\right)(a - x)^6$ is -20, find in exact form the possible values of the constant a. [5]

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8 Functions f and g are defined as follows:

$$\mathrm{f} : x \mapsto x^2 - 1 \text{ for } x < 0,$$
$$\mathrm{g} : x \mapsto \frac{1}{2x+1} \text{ for } x < -\tfrac{1}{2}.$$

(a) Solve the equation $\mathrm{fg}(x) = 3$. [4]

(b) Find an expression for $(\mathrm{fg})^{-1}(x)$. [3]

9 **(a)** A geometric progression is such that the second term is equal to 24% of the sum to infinity.

Find the possible values of the common ratio. [3]

(b) An arithmetic progression P has first term a and common difference d. An arithmetic progression Q has first term $2(a + 1)$ and common difference $(d + 1)$. It is given that

$$\frac{\text{5th term of } P}{\text{12th term of } Q} = \frac{1}{3} \quad \text{and} \quad \frac{\text{Sum of first 5 terms of } P}{\text{Sum of first 5 terms of } Q} = \frac{2}{3}.$$

Find the value of a and the value of d. [6]

10 Points $A\,(-2,\ 3)$, $B\,(3,\ 0)$ and $C\,(6,\ 5)$ lie on the circumference of a circle with centre D.

(a) Show that angle $ABC = 90°$. [2]

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(b) Hence state the coordinates of D. [1]

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(c) Find an equation of the circle. [2]

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The point E lies on the circumference of the circle such that BE is a diameter.

(d) Find an equation of the tangent to the circle at E. [5]

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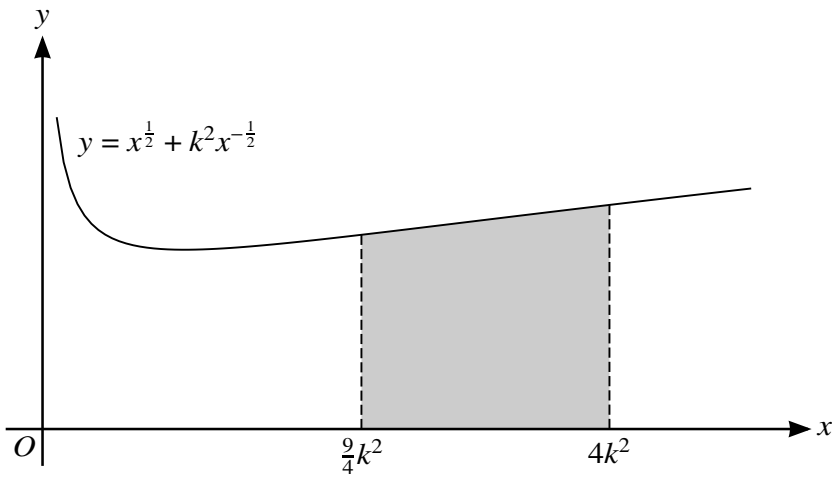

The diagram shows part of the curve with equation $y = x^{\frac{1}{2}} + k^2x^{-\frac{1}{2}}$, where k is a positive constant.

(a) Find the coordinates of the minimum point of the curve, giving your answer in terms of k. [4]

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The tangent at the point on the curve where $x = 4k^2$ intersects the y-axis at P.

(b) Find the y-coordinate of P in terms of k. [4]

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The shaded region is bounded by the curve, the x-axis and the lines $x = \frac{9}{4}k^2$ and $x = 4k^2$.

(c) Find the area of the shaded region in terms of k. [3]

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Additional Page

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